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ANGULAR MOMENTUM REVIEW

IMPORTANT RELATIONSHIPS

$\Sigma L_1 + \Sigma \tau_{ext} \Delta t = \Sigma L_2$	$\lvert L \rvert = r \, X \, p = rmvsin\theta$	$\lvert L \rvert = I\omega$

SUMMARY:

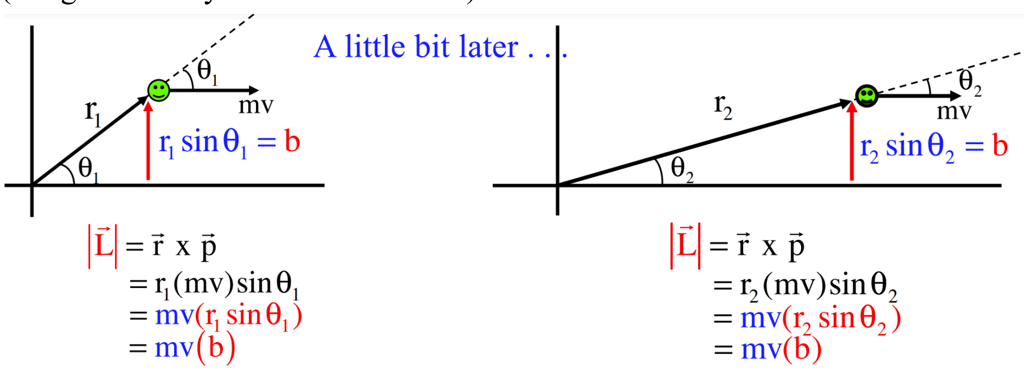
This unit deals with angular momentum. Which is a measurement of a nonexistent thing that tells you the quantity of rotation for a particular object. We will go over when angular momentum is conserved and how to tell if there is an external torque when doing a problem.

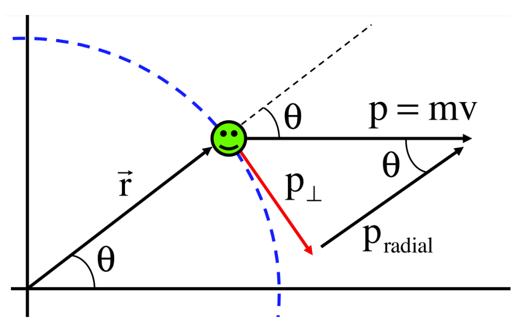
DEFINITIONS:

Angular momentum: The amount of torque required to make an object stop rotating. It is a fictitious quantity, which means it is something that we made up to quantify the phenomenon we observe in the natural world.

IMPORTANT NOTES:

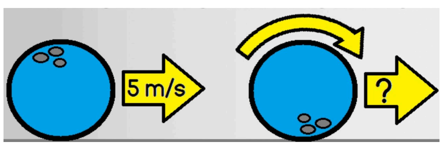
It is important to remember that even objects moving in straight lines have angular momentum. This is due to the fact that the object's angular momentum can be broken down into components. You can then use $\lvert L \rvert = r \, X \, p\perp$ where $p\perp$ is the momentum of the body moving in a straight line. (Images courtesy of Fletcher's slides).



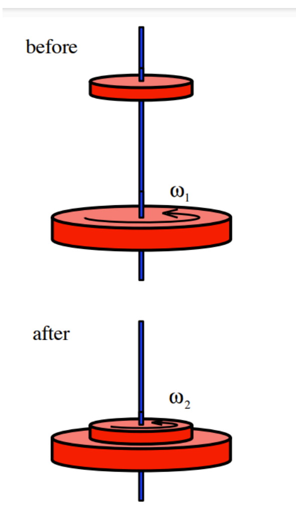


How to tell if Angular Momentum is conserved:
Angular momentum will be conserved as long as there are no external torques. An external torque is any force being applied by something not in the system that causes the object to rotate. An example of external torque would be a bowling ball traveling down an alley. At first it just slides but then the frictional force of the ground applies an external torque to the bowling ball causing it to roll. This friction is applying an external torque to the bowling ball and thus angular momentum is not conserved. An example of angular momentum being conserved is if you have one rotating plate and one stationary plate and you put the rotating plate on top of the stationary one causing them both to rotate at the same angular velocity. Since the torque of one plate causes a torque on the other plate, those are internal to the system thus there is no external torque and angular momentum is conserved.

L IS NOT CONSERVED:

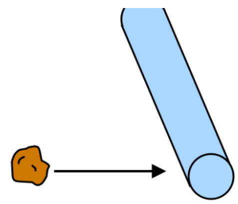


L IS CONSERVED:



Another important note: Since $|L| = I\omega$ and $\omega = v/r$ if the radius gets smaller than the angular velocity will increase but the I will decrease since there is more mass near the center of the rotating object. To prove my point go and sit on a stool and spin with your legs straight out then bring your legs in. Notice how you speed up when you bring your legs in! This is due to conservation of angular momentum. You are decreasing your I by lowering your radius, which results in a greater angular velocity! Quite fun isn't it!

PRACTICE PROBLEMS:



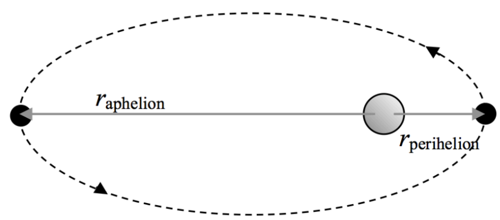
An asteroid traveling through space collides with one end of a long, cylindrical satellite as shown above, and sticks to the satellite. Which of the following is true of the isolated asteroid-satellite system in this collision?

a. Kinetic energy K is conserved
b. Total Energy E is conserved, but angular momentum $\mathbf{L}$ is not conserved
c. Angular momentum $\mathbf{L}$ is conserved, but linear momentum $\mathbf{p}$ is not conserved
d. Angular momentum $\mathbf{L}$ is conserved, and total energy E is conserved
e. Linear momentum $\mathbf{p}$ is conserved but angular momentum $\mathbf{L}$ is not conserved

SOLUTION:

Since this is an isolated system all three of the conservation laws are in effect. Linear and Angular momentum are conserved as well as mechanical energy. Thus the answer is D.

Question:



a. $v_{perihelion} < v_{aphelion}$, because the angular momentum of the system has decreased
b. $v_{perihelion} < v_{aphelion}$, because the angular momentum of the system is the same
c. $v_{perihelion} > v_{aphelion}$, because the angular momentum of the system has increased
d. $v_{perihelion} > v_{aphelion}$, because the angular momentum of the system is the same
e. $v_{perihelion} = v_{aphelion}$, because the angular momentum of the system is the same

SOLUTION:

Because angular momentum is conserved we can write the following:

$$L_i = L_f$$

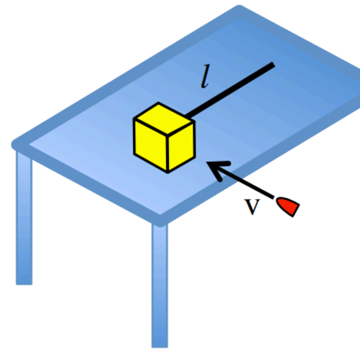
$$r_{aphelion} X \, mv_{aphelion} = r_{perihelion} X \, mv_{perihelion}$$

Because r and v are perpendicular we can say r X v = rv. Thus,..

$$v_{perihelion} = \frac{r_{aphelion}}{r_{perihelion}} v_{aphelion}$$

Given the equation above and the fact that we know that the radius of aphelion is greater than the radius of perihelion we can conclude that the velocity at perihelion is greater! The answer is D!

11.37) Situated on a frictionless tabletop is a wooden block of mass M attached to a massless rod of length l that is, itself, attached to a pivot at its other end (it would be more satisfying to use L for the length, but L is the symbol we are using for angular momentum, and that's going to get confusing). A bullet of mass m moving with velocity magnitude v strikes the block square (that is, in the horizontal, parallel to the table) and embeds itself into the block.



a.) Determine the block/bullet system's initial angular momentum, relative to the pivot.

b.) What percentage of mechanical energy is "lost" to heat and deformation and sound (i.e., to "internal" energy) through the collision?

SOLUTION

Pt a.) remember object moving in a straight line have L!

$$L_o = L_{slug}$$
$$= \vec{r}_{slug} \times (m_{slug}\vec{v})$$
$$= mvl\sin(90°)$$
$$\boxed{L_o = mvl}$$

Pt b.) To find lost KE you need the Final angular speed! treat the slug as a point mass.

$$\Sigma L_{before\ collision} + \Sigma \tau \Delta t^{\,0} = \Sigma L_{after\ collision}$$
$$\vec{r} \times (m_{slug}\vec{v}) = (I_{block} + I_{slug})\,\omega$$
$$mvl = (Ml^2 + ml^2)\,\omega$$
$$\omega = \frac{mv}{(M+m)l}$$

Finding the Fraction:

$$\frac{KE_{Final}}{KE_{initial}} \rightarrow \frac{\frac{1}{2} I_{tot}\,\omega^2}{\frac{1}{2} m_{slug} v^2} \rightarrow \frac{\frac{1}{2}(Ml^2+ml^2)\left[\frac{mv}{(M+m)l}\right]^2}{\frac{1}{2} m v^2} \rightarrow \frac{\frac{1}{2}\left(\frac{m^2}{(M+m)}\right)v^2}{\frac{1}{2} m v^2}$$

$$\frac{\frac{1}{2}\left(\frac{m^2}{M+m}\right)v^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} \Rightarrow \frac{\frac{1}{2}mv^2\left[\left(\frac{m}{M+m}\right) - 1\right]}{\frac{1}{2}mv^2} \Rightarrow \left(\frac{m}{M+m}\right) - 1$$

$$\left(\frac{m}{M+m}\right) - \frac{(M+m)}{(M+m)} = \boxed{\frac{M}{M+m}}$$